Intervalley scattering in n type Ge



Figure 2. Normalized Hall constant  $(R_{\rm H}/R_{\rm H_0})$  and Hall mobility  $(\mu_{\rm H}/\mu_{\rm H_0})$  to 65 kbar in n type Ge at 295 K;  $\triangle$  for  $\langle 100 \rangle$  and  $\bigcirc$  for  $\langle 111 \rangle$  orientations. Results converge above 30 kbar indicating hydrostatic conditions. Also  $R_{\rm H}/R_{\rm H_0}$  returns to the original normalized value by 60 kbar, and  $\mu_{\rm H}(60)$  is 850 cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup>.

determined from measurements on the elastic constants by Bond *et al.* (1950). It is seen that  $R_{\rm H}/R_{\rm H_0}$  passes through the characteristic band transfer maximum near band cross-over. This effect has been seen before in transfers from high to low mobility bands in several materials (Pitt and Lees 1970). The curves for both orientations in  $R_{\rm H}/R_{\rm H_0}$  return to the original normalized value, within experimental error, by 65 kbar and level off. This implies that transfer has been fully completed with no carrier loss to deep lying impurity levels which might exist at atmospheric pressure above the L<sub>1</sub> minima. Such resonance states have been observed in several III–V compounds (Paul 1968). Perhaps the fact that they have not been observed in Ge to date reflects the advanced state of the technology in growing the crystals (ie the existence of these nonhydrogenic levels in the III–V compounds may be associated with crystal defects).

The Hall mobility falls gradually at low pressures probably because of an increase in effective mass as the  $L_1$  band moves away from the valence band. Before band cross-over the drop in mobility is largely due to intervalley scattering between the  $L_1$  and  $\Delta_1$  minima and also transfer to the lower mobility band. We note that the mobility does not pass through a minimum at band cross-over. This is because maximum scattering between the minima will occur when the  $L_1$  minima are lower in energy than the  $\Delta_1$  minima owing to the smaller density of states in the  $L_1$  minima.

## 4. Analysis of Hall and resistivity data

The formulation of the equations including the scattering and electrical parameters will be described first, followed by an analysis of the assumptions that have been made. We use the same symbols, where possible, as employed by Nathan *et al.* (1961), and con-

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sider only the two sets of minima. The  $L_1$  minima situated in energy  $\Delta E_0$  below the  $\Delta_1$  minima. Electrons occupying the  $L_1$  minima are represented by the subscript g and  $\Delta_1$  by the subscript s.

For g electrons we may write the scattering time in the form

$$\frac{1}{\tau_{g}(E)} = A_{g}C'_{g}E^{\frac{1}{2}} + B_{g}C'_{s}(E - \Delta E)^{\frac{1}{2}}\nu_{s} \qquad E \ge \Delta E$$
$$= A_{g}C'_{g}E^{\frac{1}{2}} \qquad E < \Delta E \qquad (1)$$

and similarly for the s electrons

$$\frac{1}{\tau_{\rm s}(E)} = A_{\rm s}C'_{\rm s}(E - \Delta E)^{\frac{1}{2}} + B_{\rm s}C'_{\rm g}E^{\frac{1}{2}}v_{\rm g} \qquad \Delta E \ge 0$$
(2)

where

 $A_{g}$  and  $A_{s} = \frac{2\pi}{\hbar} |$  equivalent intervalley or intravalley scattering matrix in the g and s minima respectively  $|^{2}$ ,

$$B_{\rm g}$$
 and  $B_{\rm s} = \frac{2\pi}{\hbar} |\text{nonequivalent intervalley scattering matrix element}|^2$ 

and

$$C'_{\rm x} = m_{\rm fx}^{\frac{1}{2}} m_{\rm tx} \, 4\pi \, \times \, \frac{2^{\frac{1}{2}}}{h^3}$$

where x refers to either g or s.

The zero point energy is taken at the g band edge, but when  $\Delta E < 0$  (ie the  $\Delta_1$  valleys lie below the  $L_1$  valleys) then the subscripts must be interchanged and the zero of energy taken at the s band edge.

The electron mobilities in the two valleys are given by (Herring 1955)

$$\mu_{g} = \frac{e \langle \tau_{g}(E) \rangle}{3} \left( \frac{1}{m_{ig}} + \frac{2}{m_{ig}} \right)$$

$$\mu_{s} = \frac{e \langle \tau_{s}(E) \rangle}{3} \left( \frac{1}{m_{s}} + \frac{2}{m_{is}} \right)$$
(3)

where

$$\langle \tau_{\mathbf{x}}^{n}(E) \rangle = \frac{4}{3\pi^{1/2}} \int_{0}^{\infty} \tau_{\mathbf{x}}^{n}(y) \, y^{3/2} \exp\left(-y\right) \mathrm{d}y$$
 (4)

where y = E/kT and Maxwell-Boltzmann statistics are assumed.

The two carrier resistivity as a function of pressure is

$$\rho(P) = \frac{1}{\sigma(P)} = \frac{1}{e\{n_{g}(P)\,\mu_{g}(P) + n_{s}(P)\,\mu_{s}(P)\}}$$
(5)

where  $n_g$  and  $n_s$  are the number of electrons in the g and s minima, and hole conduction has been neglected. The  $\mu_x(P)$ , where x refers to either g or s, may be expressed in terms of a mobility associated with equivalent intervalley and intravalley process only,  $\mu_x^*(P)$ . The resulting relaxation time may be obtained directly from equation (1) by allowing  $\Delta E$  to go to infinity, in which case only these processes contribute to  $\tau_g(E)$ . Hence from equation (3) we can write

$$\frac{\mu_{g}(P)}{\mu_{g}^{*}(P)} = \frac{\langle \tau_{g}(\Delta \epsilon) \rangle}{\langle \tau_{g}(\infty) \rangle}$$
(6)

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